are shown from the south nearly to the north end of the peninsula. The geological map shows the rock formations along half of the west coast, and in a broad strip through the centre of the country. The places where gold, copper, iron, sulphur, and coal were discovered are indicated.

AFRICA.

NATAL.—Map of Natal showing mine department districts. Scale, 20 statute miles to an inch. Report on the Mining Industry of Natal for 1902. Pietermaritzburg, 1903.

This map supplements a very full report on the mining industry of the colony. It shows the distribution of the mineral deposits, in which respect Natal is highly favoured, and particularly in the extent of its coal measures.

MR. G. W. LITTLEHALES' GRAPHIC SOLUTION OF THE ASTRONOMICAL TRIANGLE AND SOME OF ITS VARIED APPLICATIONS TO THE PROBLEMS OF NAVIGATION.

BY

J. W. FROLEY.

A paper read before the Philosophical Society of Washington, in October, 1903, by Mr. G. W. Littlehales, entitled "A New and Abridged Method of Finding the Locus of Geographical Position and the Compass Error," outlines a work prepared by him which has for its aim the lightening of the labours of every ocean navigator adopting it. It is well known to mariners, and all others as well who have given attention to the subject, that the life of the navigator is absorbed in the tedium of arithmetical processes involving the rules of logarithms applied to the formulæ of spherical trigonometry. When his burdens and responsibilities are considered, it is needless to say that any means for rendering the task of the navigator easier is not only a boon to himself but an advance in the maritime interests of the world.

An important problem in navigation is the laying down accurately on the chart the Sumner Line of Position. It requires the accurate determination of the altitude and azimuth of a celestial body due to an estimated position of the observer. The navigator, having measured the true altitude of a celestial body and then computed the altitude and azimuth of the observed body due to the estimated geographical position of the ship, can draw a line upon his chart through this estimated geographical position of the ship at right angles to the azimuth or true bearing of the observed

celestial body, which is denominated the Sumner Line of Position by Account; and next comparing the observed true altitude with the altitude due to the estimated geographical position, he is enabled at once to draw the actual Sumner Line of Position, it being sensibly parallel to the Sumner Line of Position by Account and at a distance from it equal to the difference in minutes of arc between the observed and deduced altitudes and in a direction toward or away from the observed celestial body, according as the true altitude by observation is greater or less than the altitude computed from the estimated geographical position.

The ordinary method of deducing the required data in the foregoing problem involves the solution of the following equations:

$$\left.\begin{aligned} \sin\frac{^2x}{2} &= \sin\frac{^2t}{2}\cos L\cos d\sec(L-d) \\ \cos Z &= \cos(L-d)\cos x \end{aligned}\right\} \qquad (1)$$

where x = an auxiliary angle,
t = the hour angle,
L = the latitude,
d = the declination,
and z = the zenith distance;

$$\sin Az = \sin t \cos d \operatorname{cosec} z \qquad (2)$$

where Az = azimuth or true bearing of the observed celestial body.

An inspection of these equations shows that their solution requires the opening of the logarithmic tables ten or twelve times, and the liability of committing numerical errors is no small factor to contend with.

Mr. Littlehales has constructed a stereographic meridian projection of the celestial sphere, and made a device of it for the solution of all spherical triangles having given two sides and their included angle. The projection is 12 feet in diameter, and is overlaid with parallels and meridians; and also a series of equally-spaced concentric circumferences, and another series of equally-spaced radial lines have been drawn over it in lines of dashes. The solution of the astronomical triangle, as given by him, is as follows:

If the latitude of the observer be laid off on the bounding meridian, and the declination of the observed celestial body be laid off along the meridian making an angle with the bounding meridian equal to the hour angle of the observed celestial body, an astronomical triangle will be formed in which the known parts are respectively the co-latitude and co-declination, and their included angle, which is the hour angle of the observed body. Two of the

unknown parts of this triangle are the azimuth and the co-altitude of the observed celestial body. If this triangle is now revolved about the central point of the projection, with the side joining the latitude of the observer and the pole kept in coincidence with the bounding meridian until the position occupied by the observer reaches the pole, the former position occupied by the pole would be on the bounding meridian, and the position occupied by the observed celestial body would fall in such a position that the unknown side of the triangle representing the co-altitude would lie along some meridian, and could be measured from the graduation of the projection, and the unknown angle representing the azimuth would become an included angle between two meridians, which could likewise be measured from the graduation of the projection; and thus the altitude and the azimuth of any observed celestial body could be simultaneously determined from the diagram with a degree of precision depending only on the scale of the projection. It may be noted here that if the altitude and azimuth are given, the declination and hour angle may be determined in a very similar manner.

The projection, being 12 feet in diameter, is too large for ordinary use when kept in one continuous sheet, so Mr. Littlehales made each quadrant into about 85 overlapping sections, all conveniently indexed; and the equally-spaced concentric circumferences and the equally-spaced radial lines, all being appropriately numbered, obviate the necessity for actual revolution of the triangle. For the position of the observed body revolves into another position, which is fixed by the intersection of the circumference that the original position was on, and a radial line determined by reducing the co-latitude to minutes and adding it algebraically to the number of the radial line passing through the original position of the revolved point.

The writer, in testing this graphic method of solution, found that the same results could be obtained in about one-fifth of the time that would be required by use of the formulæ.

The method lends itself with equal facility to the determination of course and distance in great-circle sailing.

The altitude and azimuth of a star having been observed through a rift of clouds may have its identity established with facility and dispatch—first, by finding its declination and hour angle, and thence its right ascension, and, second, by reference to a star catalogue.

It is believed that this graphic method opens a way by which all mariners may practice the science of astronomical navigation, which has heretofore been restricted to those of higher education.